Adhesion dynamics of confined membranes:

a simple model

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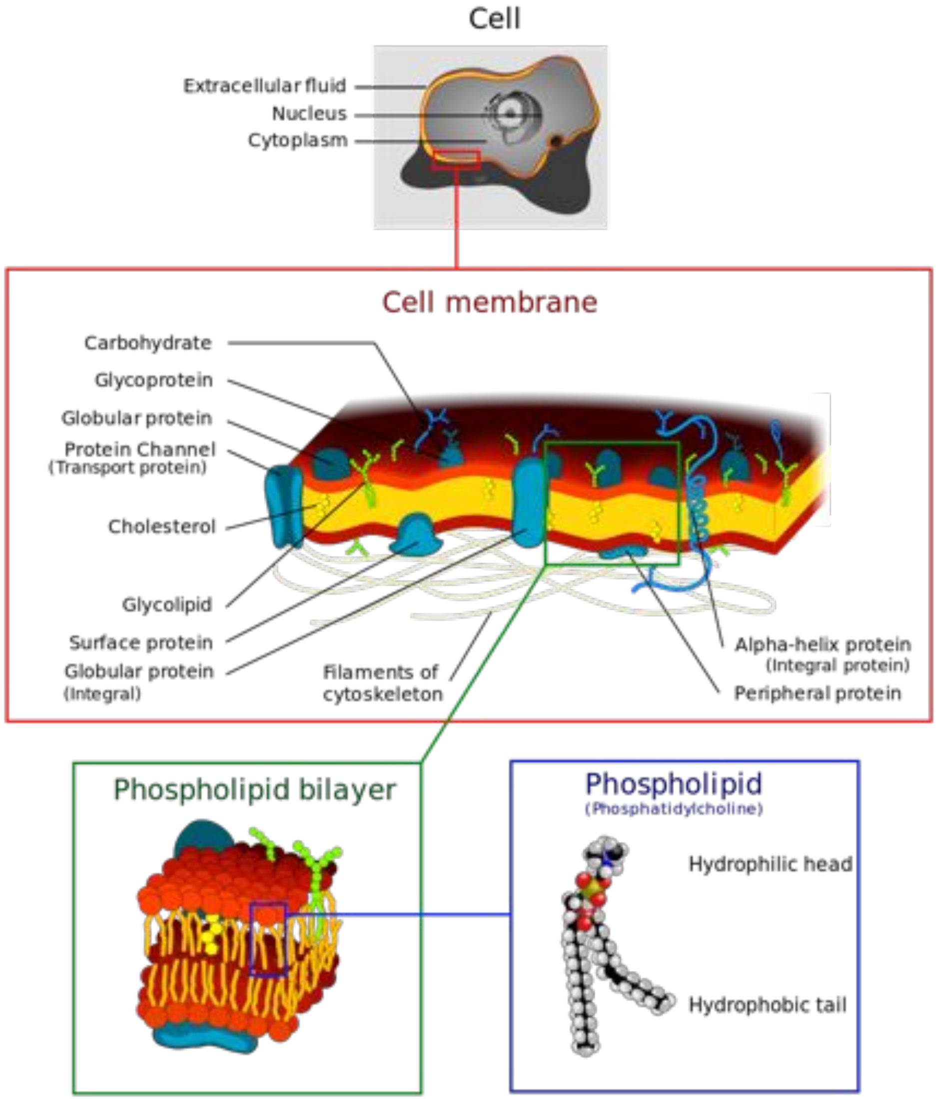
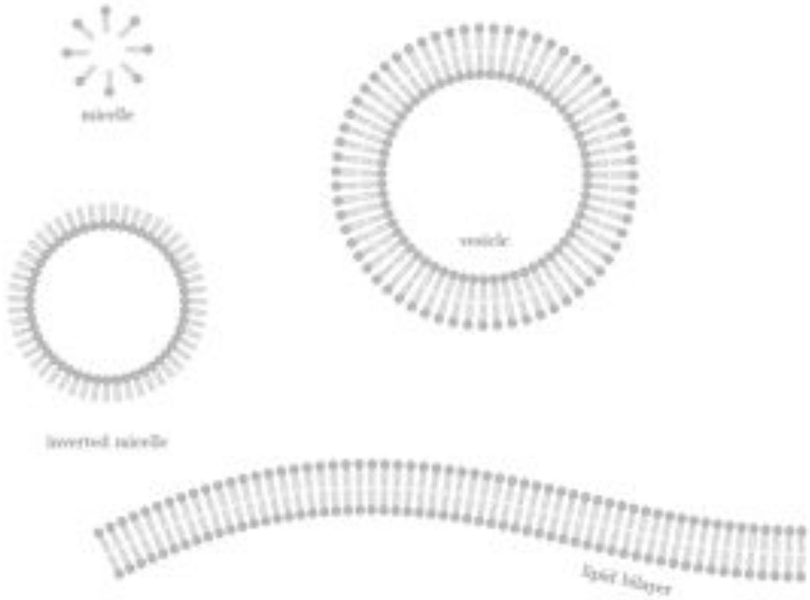
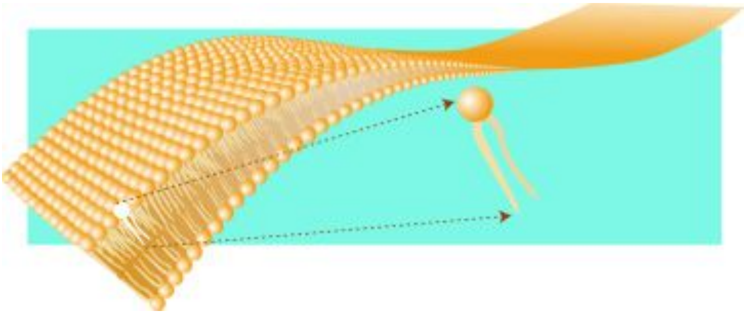
Most famous, most complex biological membrane



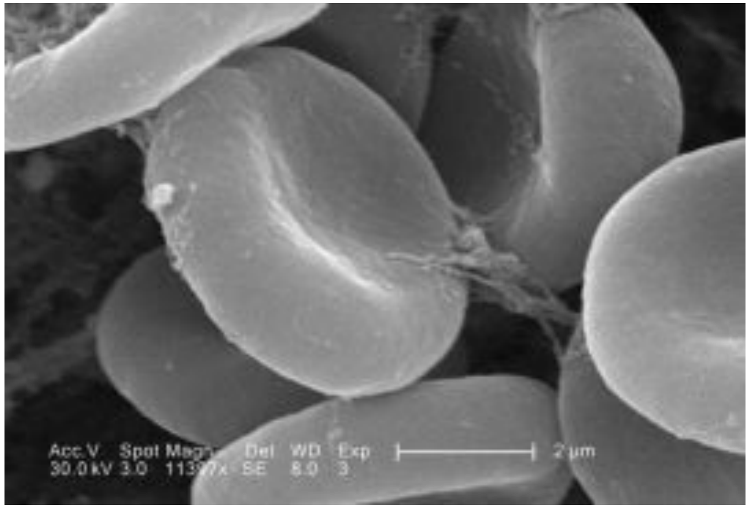
Most manegeable:
a lipid bilayer



Isolated membrane



- Bending energy,
  not surface tension energy
- The membrane is in a fluid state
  (no shear modulus)
- Almost no stretching



- Biconcave shape of red blood cells

Interaction between (rigid) membranes

Short range repulsion

(Hydration or entropic forces)

$+$

Long range attraction

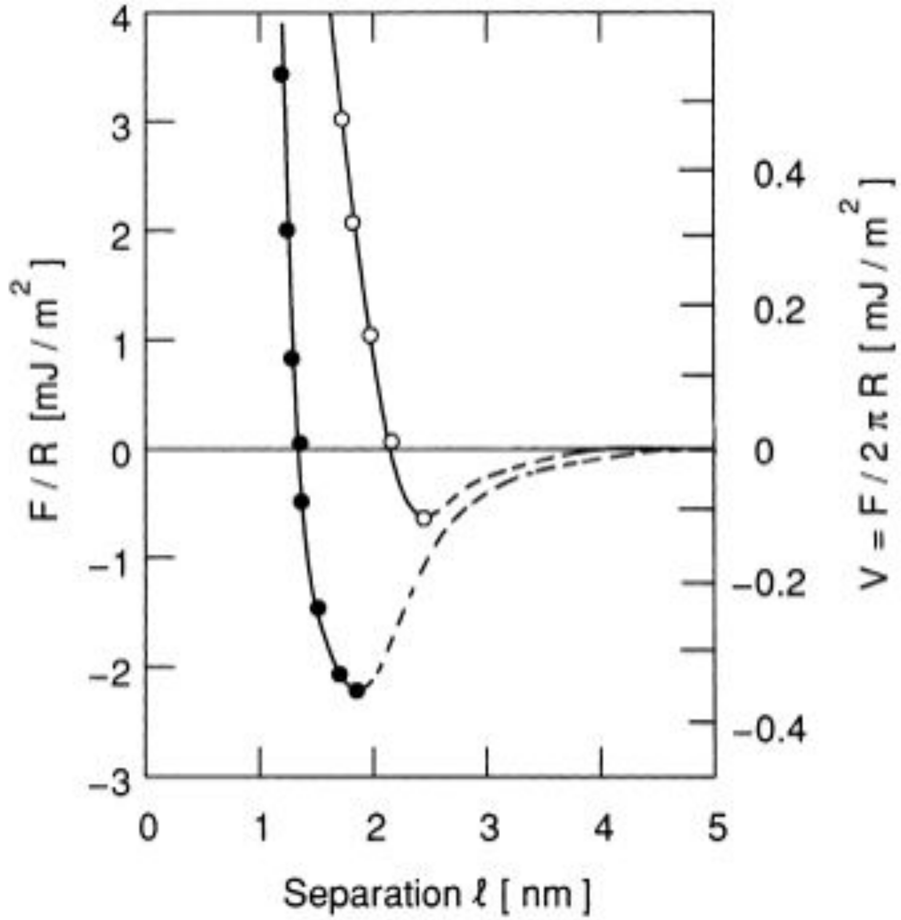
(Van der Waals forces)

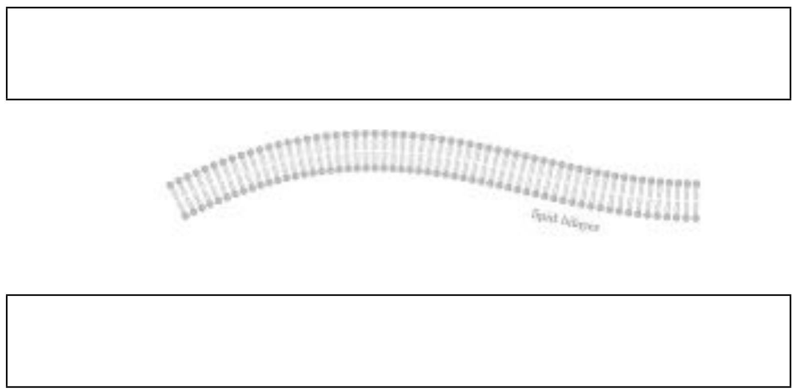
$\Downarrow$

Adhesion

(to another membrane

or to a substrate)

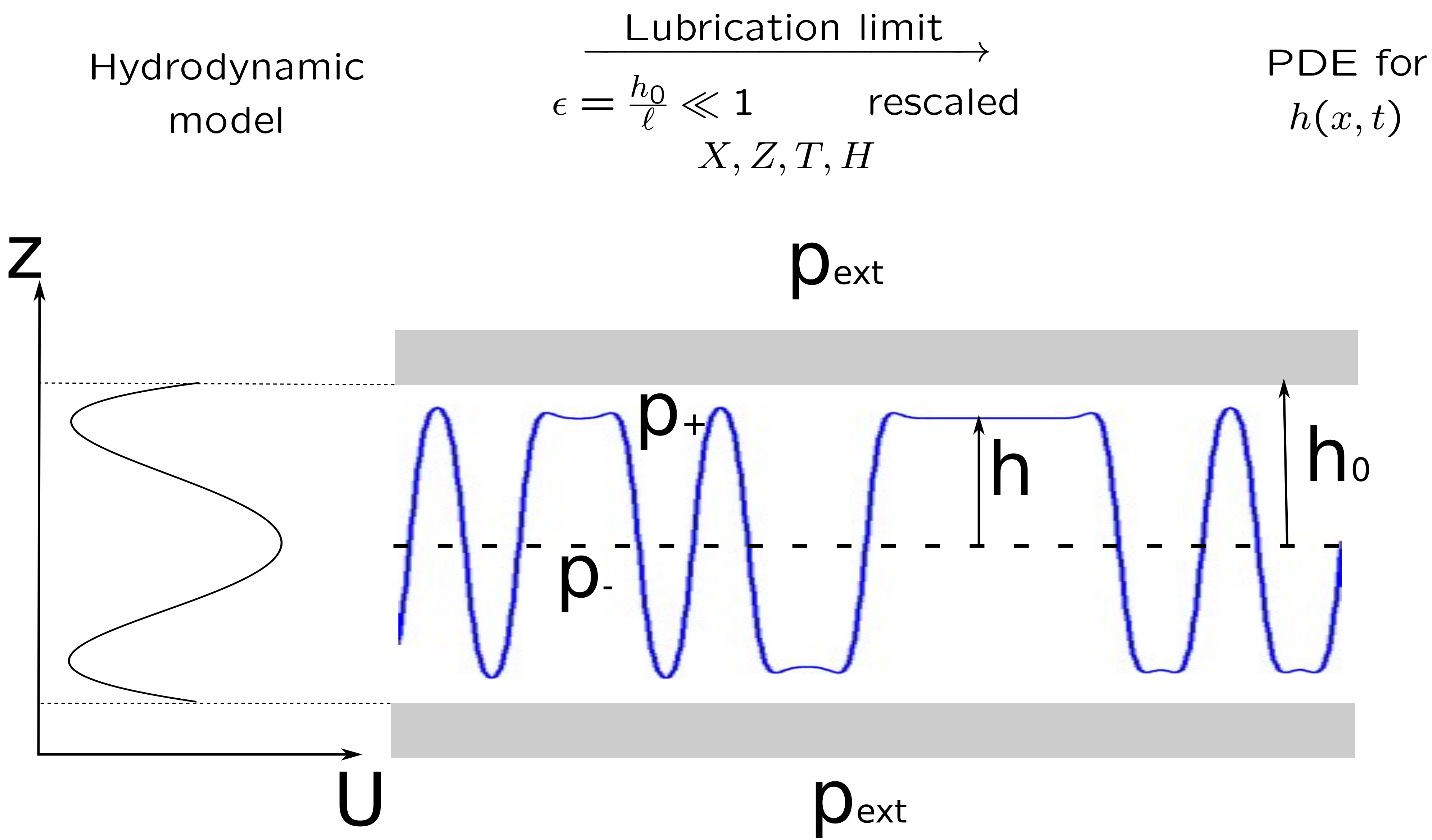




Our goal:

Study the adhesion dynamics of a

confined membrane


Hydrodynamic model
Lubrication limit
$\epsilon = \frac{h_0}{\ell} \ll 1$
rescaled
X, Z, T, H
PDE for $h(x,t)$
z
U
p_{ext}
p_+
p_-
h
h_0
p_{ext}

$\bar{\nu} \equiv$ reduced wall permeability

$$\boxed{\partial_T H(X,T) = -\partial_X^4 H - U'(H)} \qquad \bar{\nu} \to \infty \text{ (TDGL4)}$$

$$\boxed{\partial_T H(X,T) = \partial_X \left\{ (1-H^2)^3 \partial_X \left(\partial_X^4 H + U'(H) \right) + J H \left(\frac{H^2}{3} - 1 \right) \right\}} \qquad \bar{\nu} \to 0$$

$$J \equiv \frac{9}{L} \int_0^L dX\, H \left(\frac{H^2}{3} - 1 \right) \partial_X \left(\partial_X^4 H + U'(H) \right) \quad \text{(non local CH4)}$$

∂_X^2 (surface tension) $\Longrightarrow$ ∂_X^4 (bending energy)

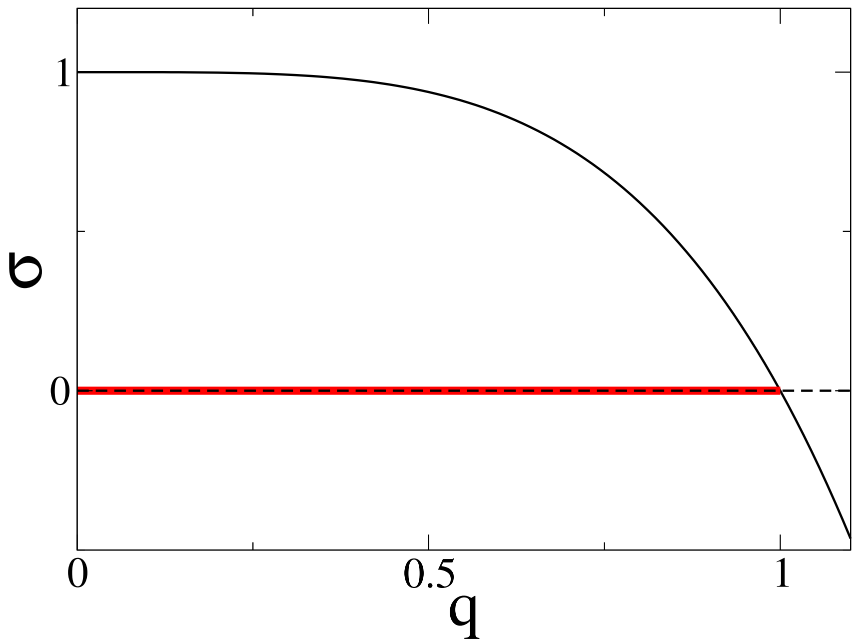
$(1-H^2)^3$ (mobility) $\xrightarrow{H \to 1} 0$

$J \iff$ Non locality

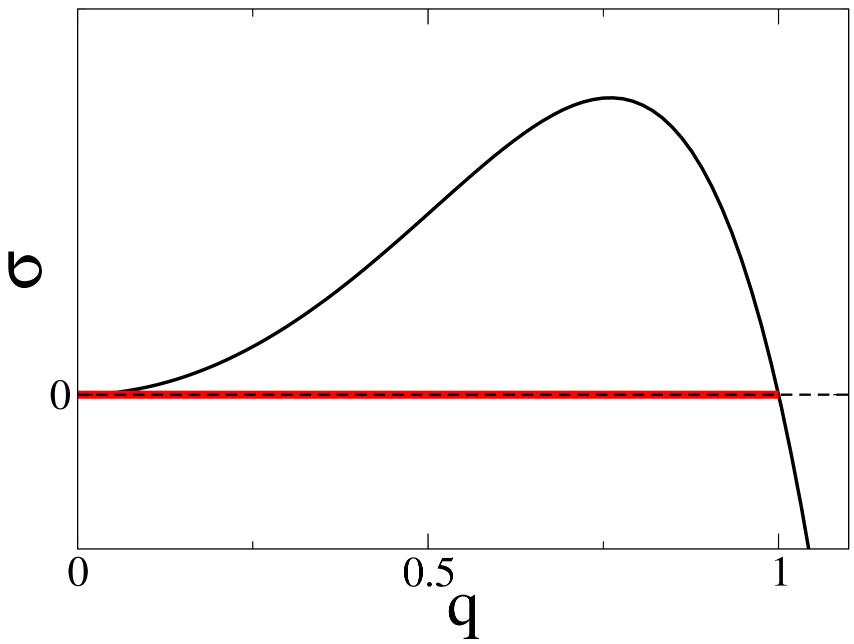
Linear behaviour

$$H(X,T) = \delta \exp(\sigma T + iqX)$$

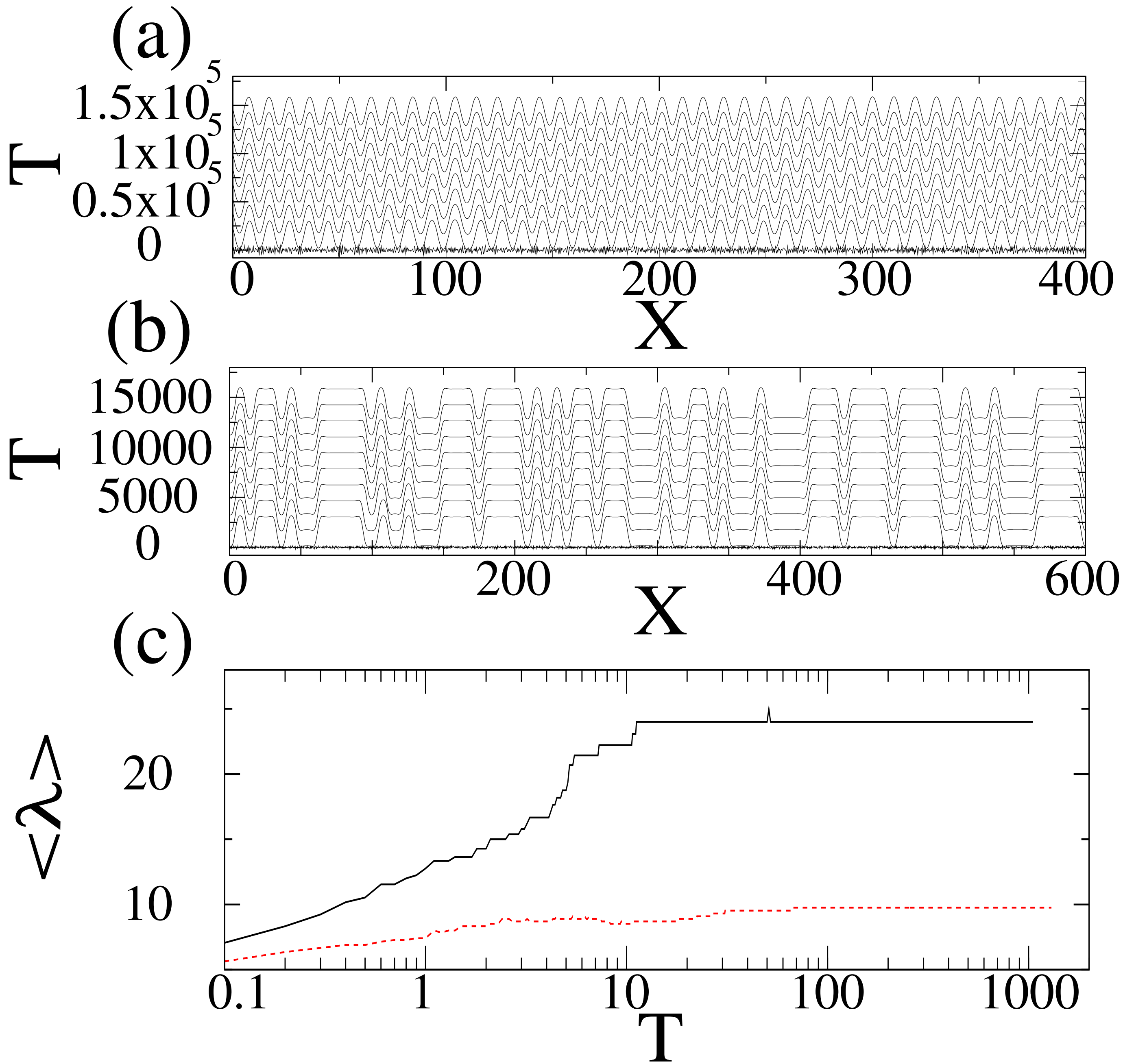
TDGL4

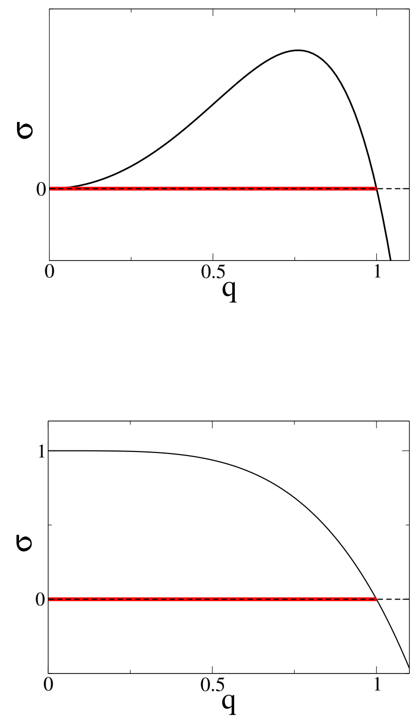


non local CH4



Nonlinear behaviour: Numerics





Nonlinear behaviour: Analitics

$$\partial_T H(X,T) = -\partial^4{}_X H - U'(H) \equiv -\frac{\delta \mathcal{F}_{\mathrm{GL4}}}{\delta H} \qquad \bar{\nu} \to \infty \text{ (TDGL4)}$$

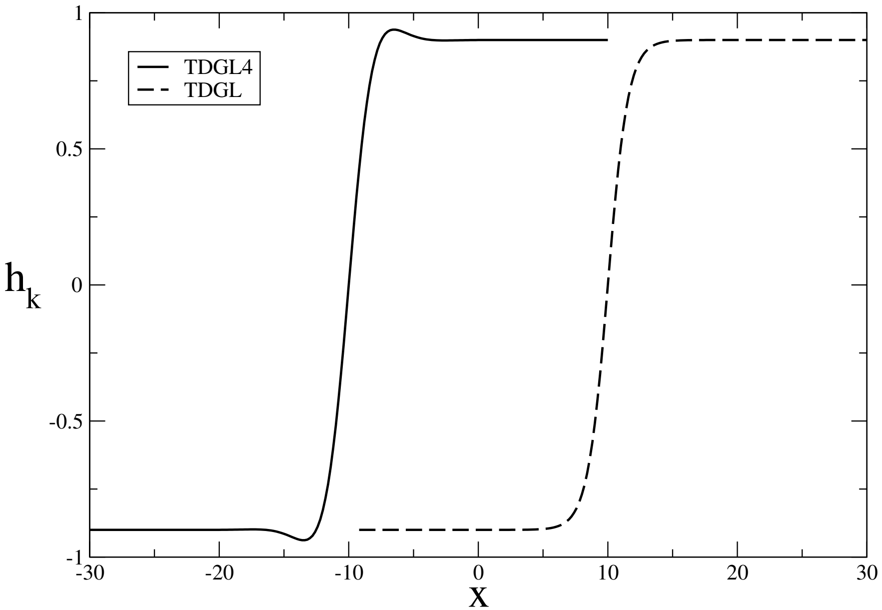
$$\partial_T H(X,T) = \partial_X^2 \left(\partial_X^4 H + U'(H) \right) \equiv \partial_{XX} \left(\frac{\delta \mathcal{F}_{\mathrm{GL4}}}{\delta H} \right) \qquad \bar{\nu} \to 0 \text{ (CH4)}$$

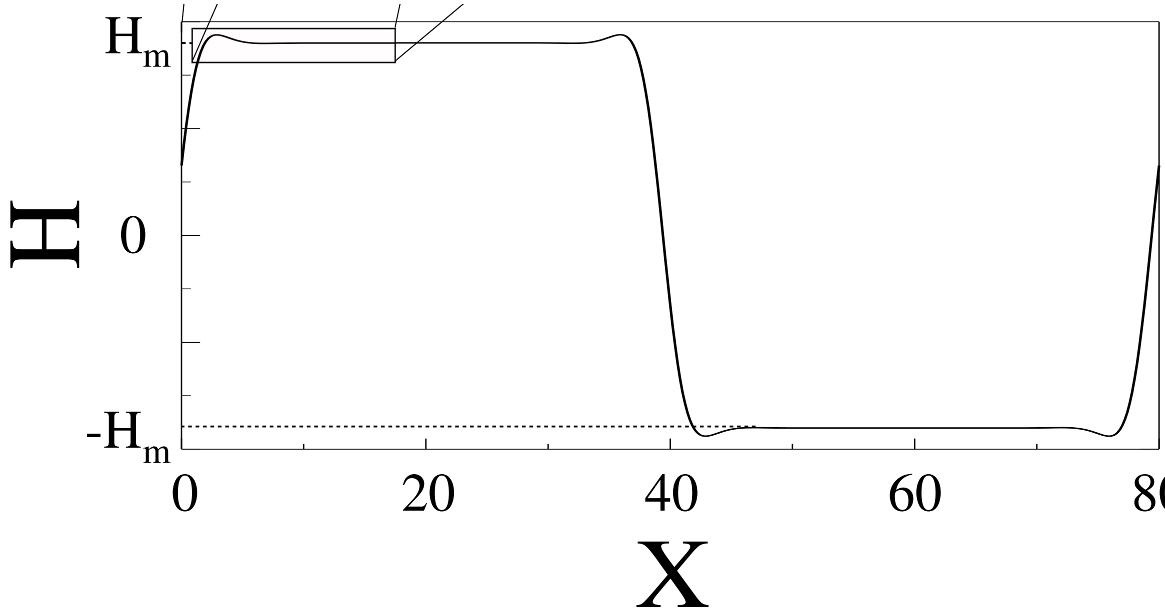
$$\mathcal{F}_{\mathrm{GL4}} = \int dX \left(\frac{1}{2} H_{XX}^2 + U(H) \right),$$

Why does $(H_X^2) \Rightarrow (H_{XX}^2)$

stops coarsening and frozen dynamics?

Because kinks, which connect the two minima, are now oscillating





and these oscillations change the stability of steady profiles

$$\mathcal{L}_\lambda = -\int_0^\lambda dX \Big(\partial_{XX} H_\lambda(X)\Big)^2 \qquad \text{stability} \quad \Leftrightarrow \quad \partial_\lambda \mathcal{L}_\lambda > 0$$

First conclusions:

- Bending freezes dynamics

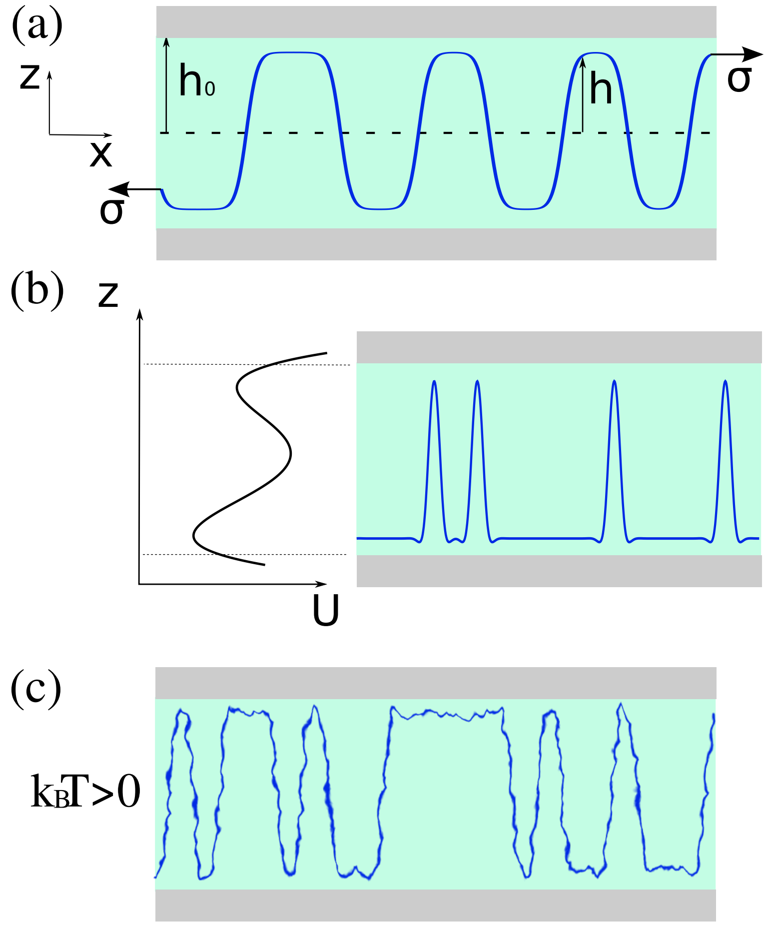
- Frozen state are more ($\bar{\nu} \to 0$) or less ($\bar{\nu} \to \infty$) disordered

How robust is the *frozen picture*?

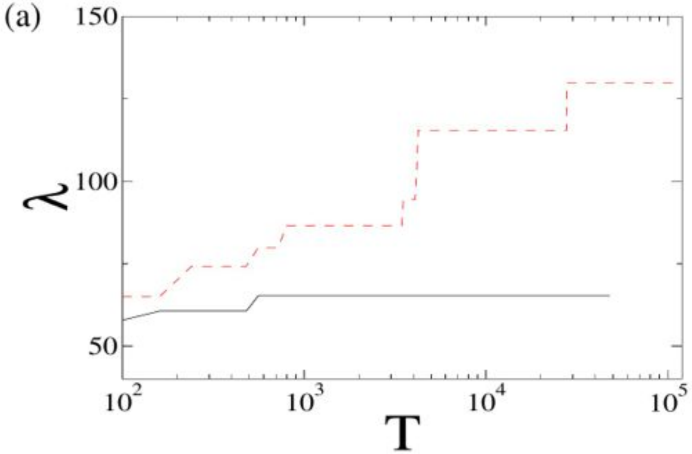
- Surface tension

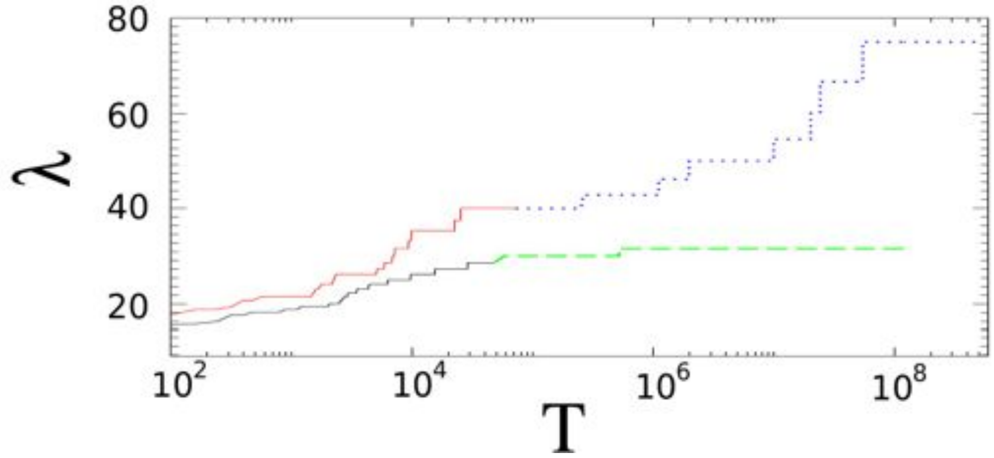
- Potential asymmetry

- Thermal fluctuations



Effect of tension (γH_X^2) ?



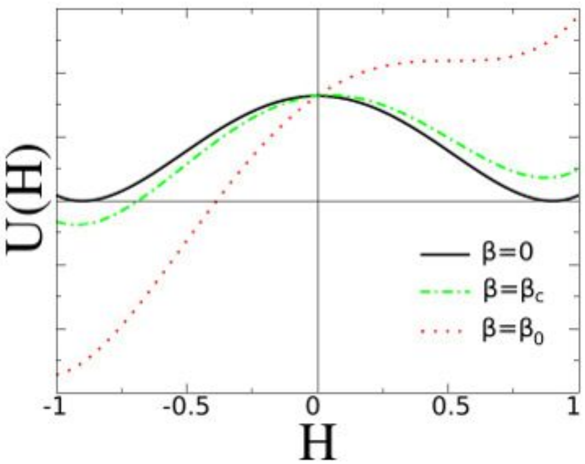


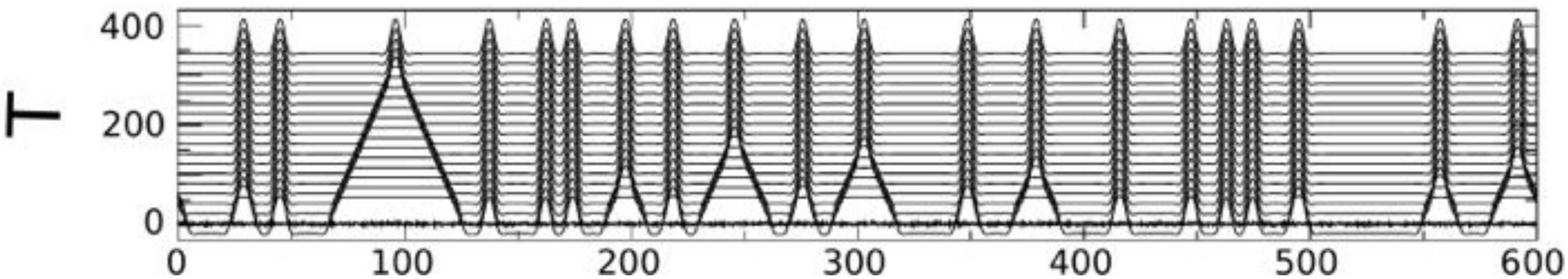
$\bar{\nu} \to \infty$

$\bar{\nu} \to 0$

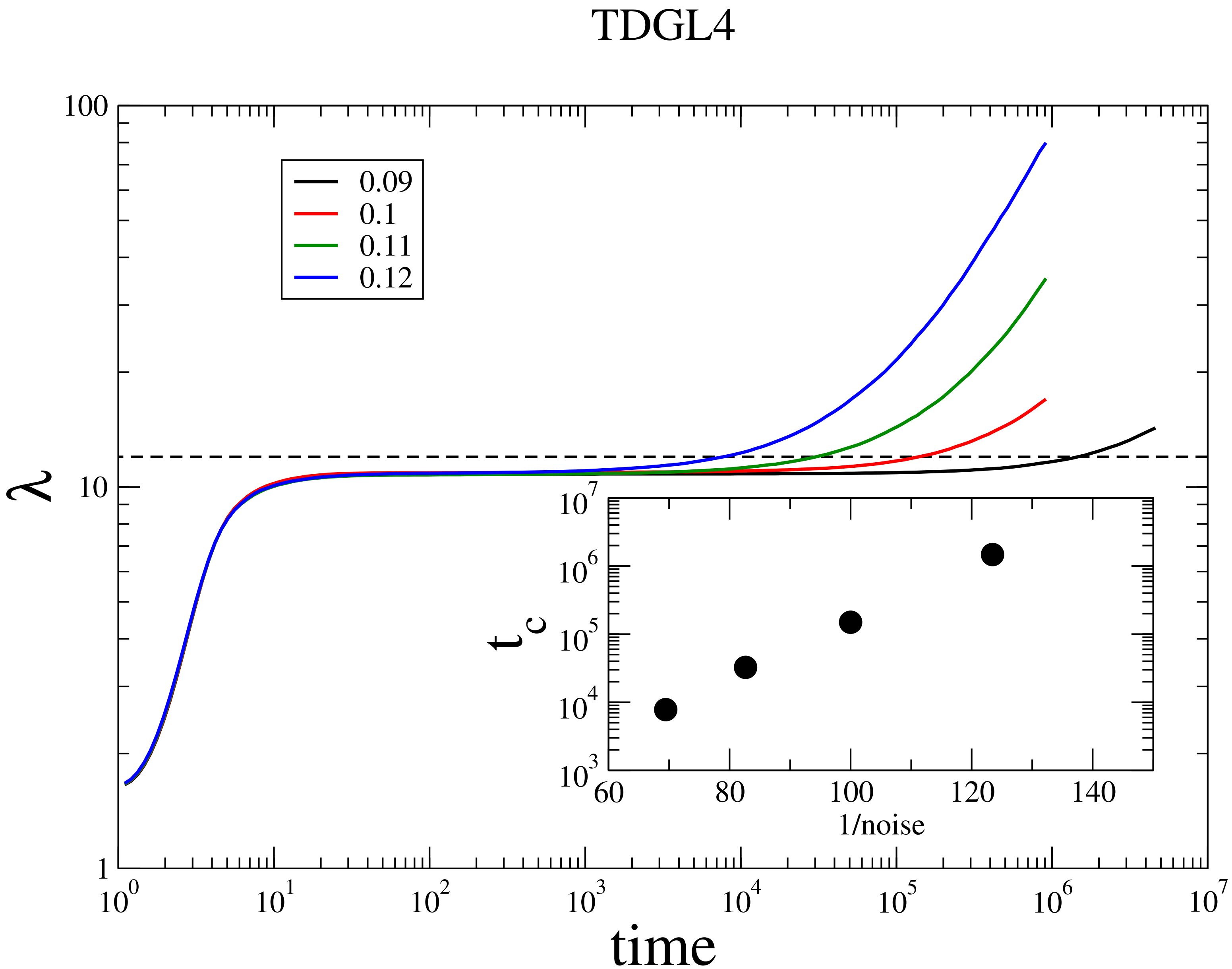
$\gamma < \gamma_c = \sqrt{4U''_m}$ oscillation persist

Effect of asymmetry? $U(H) = U_s(H) + \beta H$





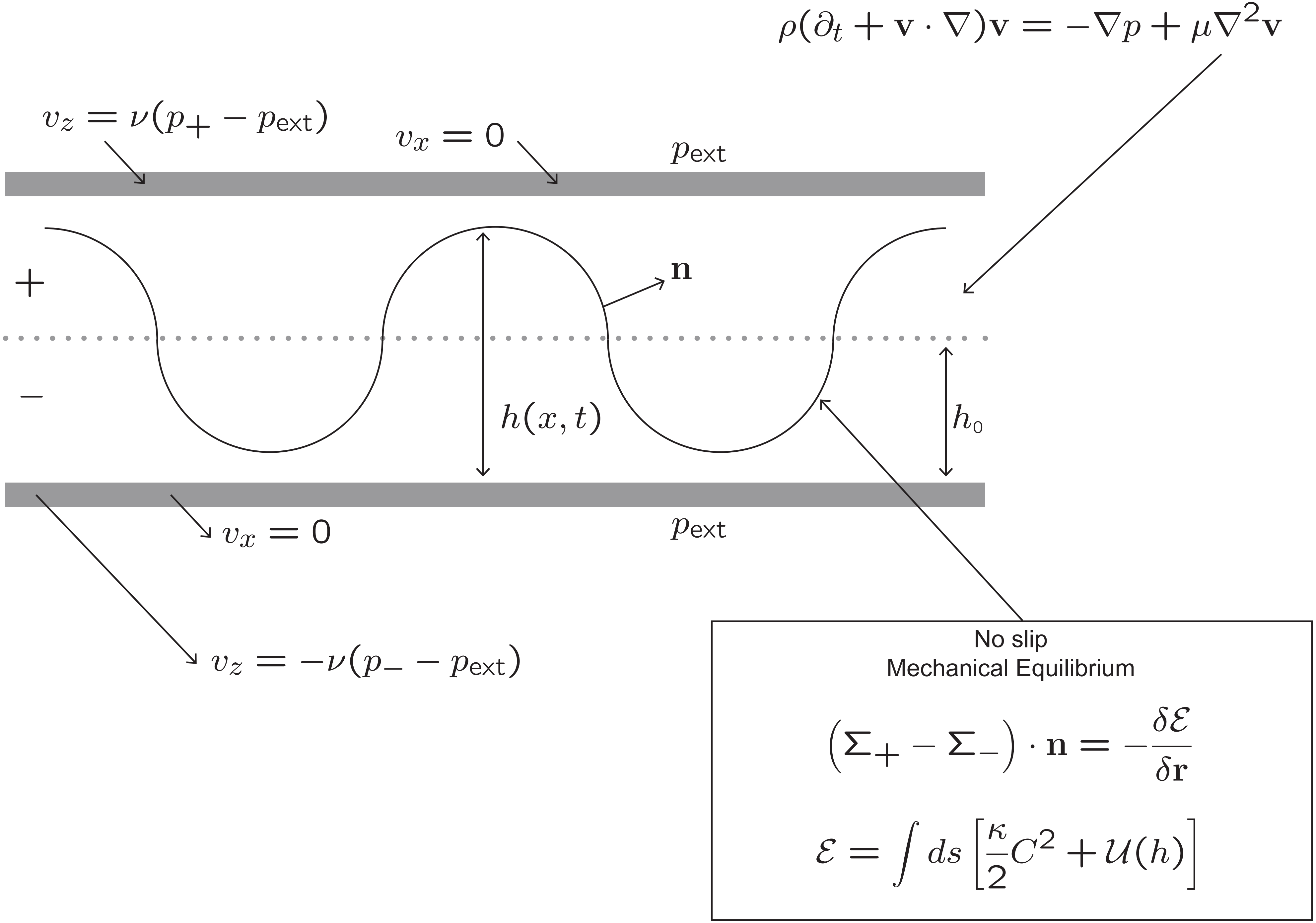
Effect of noise?



Summary

Bending + Double well potential = Frozen state

$$\text{Strong} \quad \left\{ \begin{array}{c} \text{Surface tension} \\ \text{Potential asymmetry} \\ \text{Thermal noise} \end{array} \right\} \quad \Longrightarrow \quad \begin{array}{c} \text{Transition to} \\ \text{coarsening} \end{array}$$


$\rho(\partial_t + \mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p + \mu\nabla^2\mathbf{v}$
$v_z = \nu(p_+ - p_{\text{ext}})$
$v_x = 0$
p_{ext}
$+$
$\mathbf{n}$
$-$
$h(x,t)$
h_0
$v_x = 0$
p_{ext}
$v_z = -\nu(p_- - p_{\text{ext}})$
No slip
Mechanical Equilibrium
$\left(\Sigma_+ - \Sigma_-\right)\cdot\mathbf{n} = -\frac{\delta\mathcal{E}}{\delta\mathbf{r}}$
$\mathcal{E} = \int ds\left[\frac{\kappa}{2}C^2 + \mathcal{U}(h)\right]$